

ASME B89.4.10-2021

[Revision of ASME B89.4.10-2000 (R2011)]

Methods for Performance Evaluation of Coordinate Measuring System Software

AN AMERICAN NATIONAL STANDARD



**The American Society of
Mechanical Engineers**

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**The American Society of
Mechanical Engineers**

Two Park Avenue • New York, NY • 10016 USA

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FOREWORD

Coordinate measuring systems (CMSs) rely upon software that processes coordinate data; often, this software computes fits of geometric elements to such data. The performance of these fits can vary among software packages, and in some cases can be a significant contributor to the overall uncertainty of measurement.

The purpose of this Standard is to provide guidelines for evaluating the quality of solutions generated by CMS software and to define minimal documentation requirements for software providers. This Standard is concerned with testing the behavior of algorithm implementation, not the testing of algorithms themselves. It is not the intent of this Standard to endorse or rate any computational method or system. A mechanism for generating collections of test data sets is specified. While a specific, static collection of standardized test data sets is not defined, the generating mechanism can produce several collections of similar character.

ASME B89.4.10-2021 was approved by the American National Standards Institute on July 22, 2021.

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Dimensional Metrology

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| | |
|-------------------------|---|
| Subject: | Cite the applicable paragraph number(s) and the topic of the inquiry in one or two words. |
| Edition: | Cite the applicable edition of the Standard for which the interpretation is being requested. |
| Question: | Phrase the question as a request for an interpretation of a specific requirement suitable for general understanding and use, not as a request for an approval of a proprietary design or situation. Please provide a condensed and precise question, composed in such a way that a "yes" or "no" reply is acceptable. |
| Proposed Reply(ies): | Provide a proposed reply(ies) in the form of "Yes" or "No," with explanation as needed. If entering replies to more than one question, please number the questions and replies. |
| Background Information: | Provide the Committee with any background information that will assist the Committee in understanding the inquiry. The Inquirer may also include any plans or drawings that are necessary to explain the question; however, they should not contain proprietary names or information. |

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ASME B89.4.10-2021

SUMMARY OF CHANGES

Following approval by the ASME B89 Committee and ASME, and after public review, ASME B89.4.10-2021 was approved by the American National Standards Institute on July 22, 2021.

In ASME B89.4.10-2021, the figures and tables have been redesignated based on their parent paragraph. ASME B89.4.10-2021 includes the following additional changes identified by a margin note, **(21)**.

| <i>Page</i> | <i>Location</i> | <i>Change</i> |
|-------------|-----------------|--|
| 1 | 1 | Second paragraph revised |
| 1 | 1.1 | Subparagraph (a) revised |
| 1 | 1.3 | Updated |
| 1 | 2 | (1) Definitions of <i>datum</i> and <i>least-squares fit feature</i> revised (2) Definition of <i>datum reference frame (DRF)</i> deleted |
| 2 | 3 | Revised |
| 3 | 4.1.2.1 | Subparagraph (a) revised |
| 3 | 4.1.2.2 | (1) Subparagraphs editorially redesignated (2) Subparagraph (b) added |
| 5 | 4.1.2.7 | First paragraph editorially revised, and last two paragraphs added |
| 7 | Table 4.1.2.7-1 | General Note added |
| 8 | 5.2.1.2 | Subparagraph (c) revised |
| 10 | 5.4.6 | Revised |
| 10 | 5.4.7 | Revised |
| 10 | 5.5.1 | First sentence and last paragraph revised |
| 11 | 5.5.2 | Revised |
| 11 | 5.5.4 | Penultimate sentence and last row value in the in-text table revised |
| 11 | 5.5.5 | (1) First paragraph of 5.5.5.1 revised (2) Last paragraph of 5.5.5.2 added |
| 12 | 5.5.6 | Revised |
| 13 | 5.7 | Last sentence deleted |
| 13 | 5.8 | First sentence in last paragraph deleted |
| 13 | 5.10 | Revised |
| 14 | 6.3.4.4 | Revised |
| 14 | 6.3.6.4 | Revised |
| 15 | 6.3.8 | Revised |
| 17 | A-1 | Subparagraph (b) revised |
| 17 | A-2 | (1) Subparagraph (a) added, and subsequent subparagraphs redesignated (2) First paragraph and subpara. (b) [formerly (a)] revised |

| <i>Page</i> | <i>Location</i> | <i>Change</i> |
|-------------|-------------------------|--|
| 18 | Nonmandatory Appendix B | Former Nonmandatory Appendix B deleted, and subsequent appendices redesignated |
| 18 | B-1 | (1) Designator and title added, and subsequent paragraphs redesignated (2) In paragraph after Disclaimer, “0.010 in.” revised to “0.010 mm” |
| 18 | B-5.4 | Subparagraph (c) deleted |
| 19 | B-6.1 | “0.025 in.” revised to “0.025 mm” |
| 19 | B-6.4 | Former para. C-5.4 deleted, and former para. C-5.5 redesignated as B-6.4 |
| 19 | B-7 | In first and second paragraphs, “flatness” revised to “flatness” |
| 19 | B-7.3 | Former para. C-6.3 deleted, and former para. C-6.4 redesignated as B-7.3 |
| 20 | B-8 | (a) Subparagraphs editorially redesignated (b) In subpara. (b), “99” revised to “9 999” |
| 20 | B-9 | Revised |
| 21 | C-1 | (1) Designator and title added, and subsequent paragraphs redesignated (2) Last sentence in third paragraph deleted |
| 21 | C-2 | Equation revised |
| 22 | C-2.2 | First sentence and equation revised |
| 22 | C-3 | Added |
| 23 | C-3.2 | (1) Last sentence of first paragraph revised (2) Last two equations and paragraphs deleted |
| 23 | C-4.1 | Second equation revised |
| 23 | C-4.2 | Second equation revised |
| 25 | Nonmandatory Appendix D | Former Nonmandatory Appendix E deleted, and subsequent appendices redesignated |
| 25 | D-1 | Designator and title added, and subsequent paragraphs redesignated |
| 25 | D-4.2 | Revised |
| 27 | Nonmandatory Appendix E | Updated |

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METHODS FOR PERFORMANCE EVALUATION OF COORDINATE MEASURING SYSTEM SOFTWARE

1 SCOPE

(21)

A critical issue in industrial coordinate metrology is the measurement of a work piece to assure compliance with its dimensional requirements. When using a computerized coordinate measuring system (CMS), the usual practice is to correlate computer-calculated outputs with the dimensional requirements of the workpiece. This correlation is performed by various computer routines that process dimensional coordinate data sets consisting of measurement samples of the object being evaluated.

The purpose of this Standard is to provide guidelines for evaluating the quality of solutions generated by CMS software and to define minimal documentation requirements for software providers. Additionally, this Standard gives default definitions for collections of data sets that span a variety of real-world measuring scenarios. These data sets are dependent on the fitting algorithm being tested. This Standard is concerned with testing the behavior of algorithm implementation, not the testing of algorithms themselves. Thus, the software is treated as a black box; only the input and output are observed and evaluated. It is not the intent of this Standard to endorse or rate any computational method or system.

Software performance evaluation is useful because it

- (a) allows objective validation of software
- (b) reduces the possibility of error in software application
- (c) defines a method of comparing CMS software

This Standard covers the following areas: input data, feature construction, software documentation, performance characterization, and test methodologies.

1.1 Assumptions

(21)

The assumptions inherent in this Standard are as follows:

- (a) Measurement uncertainty in coordinate samples is not addressed.
- (b) Methods to input predetermined samples to the computational system are available.
- (c) Personnel have adequate experience and training to implement the evaluation and understand the implications of the results.

1.2 Application

This Standard is one component required for the evaluation of CMSs. Other relevant documents can be found in [Nonmandatory Appendix E](#).

1.3 References

(21)

The following is a list of standards referenced in this Standard. Unless otherwise noted, the most recent edition shall apply.

ASME Y14.5, Dimensioning and Tolerancing

ASME Y14.5.1, Mathematical Definition of Dimensioning and Tolerancing Principles

Publisher: The American Society of Mechanical Engineers (ASME), Two Park Avenue, New York, NY 10016-5990
(www.asme.org)

See [Nonmandatory Appendix E](#) for additional, informative references.

2 DEFINITIONS

(21)

algorithm: a well-defined procedure for solving a particular problem, e.g., sorting algorithms.

coordinate measuring system (CMS): any piece of equipment that collects coordinates (points), calculates, and displays additional information using the measured points.

datum: a theoretically exact point, line, or plane derived from a feature on a part. See ASME Y14.5M-2018.

least-squares fit feature: a feature of perfect form, corresponding to a set of data points, that minimizes the sum of the squared deviations between the feature and the individual data points. (Reference [Nonmandatory Appendix C](#) for additional information.) This term is elsewhere sometimes referred to as the Gaussian associated feature.

NOTE: In this Standard, unless otherwise indicated, the least-squares fit is understood to be not weighted, i.e., each point is given equal weight in the least-squares objective function, even if the points in the test data are not exactly evenly spaced.

objective function: a function which is to be optimized by searching for a minimum (or maximum) as its parameters are varied. A different objective function is used for each type of fit, e.g., a least-squares versus minimum-circumscribed circle.

reference evaluation: the evaluation of the substitute feature using a known implementation of an algorithm.

reference feature: a substitute feature used as the basis for evaluating a test feature.

substitute feature: a feature of perfect geometric form that corresponds to a set of data points and is intended to minimize an objective function.

test: a basic unit of evaluation, based on one or more related data sets, which are applied to one or more software implementations of an algorithm.

test feature: a substitute feature computed by the software under test.

(21) **3 SOFTWARE FUNCTIONS**

In normal usage, CMS hardware is used to collect data points (raw data) on the surfaces of parts being inspected. CMS software can process these raw data to construct datums, part coordinate systems, and substitute features that represent the surfaces being inspected. From these constructions, the CMS software can evaluate such characteristics as size, location, orientation, and form.

3.1 Input Data

Raw data to be used to test and analyze CMS software may be obtained by physically inspecting a test workpiece or by mathematical computation. The former represents a test of the entire measuring system, while the latter approach avoids operator, workpiece, environment, and machine influences. The latter approach also makes it possible to more closely control the raw data sets, including limits on their spatial distribution, as well as inclusion of artificially induced form errors. For software analysis, the latter approach is the most universally accepted and the most reliable. This is the approach addressed herein.

3.2 Data Analysis

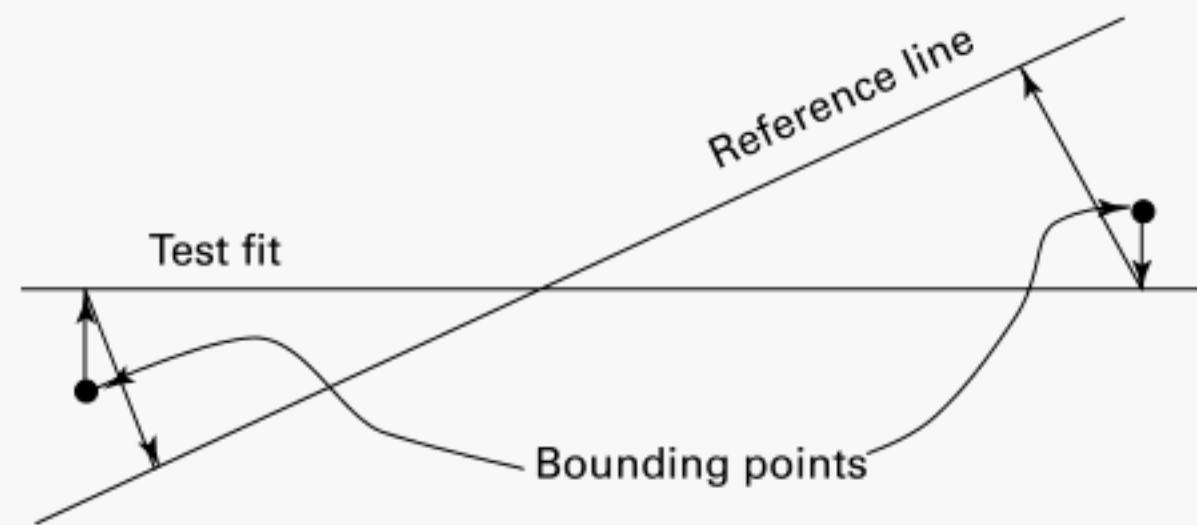
The raw data points are processed by mathematical algorithms with the purpose to calculate perfect-form substitute features. First, substitute features are calculated to represent the original data. Then the substitute features are used to evaluate conformance to tolerances or to determine other geometric characteristics of the workpiece. An alternative to the use of substitute features is the use of Functional Gage Simulation, described in [Nonmandatory Appendix D](#).

Different methods can be used for obtaining substitute features. These methods may have different objective functions, i.e., different criteria for deciding that a particular substitute feature is better or worse than other possible substitute features. Different criteria can, in general, lead to different results. The proper selection of fitting criterion and data analysis method is outside the scope of this Standard. Fit criteria are usually based on L^P -norm estimation, or minimum-circumscribed, or maximum-inscribed methods. Refer to [Nonmandatory Appendix C](#) for explanations of these methods.

The objective of this Standard is not to decree that any one method is better than any other. Guidance is provided to the user for checking whether particular CMS software produces results that agree sufficiently closely with the reference results within the context of the design requirements.

4 PERFORMANCE CHARACTERIZATION

This section establishes the characteristics by which CMS software performance is evaluated. These characteristics are discussed in terms of four categories: quality, robustness, reliability, and ease of use. Characteristics that are not used for performance evaluation in this Standard are discussed at the end of this section.

Figure 4.1.1-1 Example of Fit Bounding

4.1 Evaluation of Quality

In this Standard, the quality of the algorithm is evaluated on the basis of the geometric deviation of the test feature from a reference feature.

4.1.1 Evaluation Concept. Some features have unbounded geometry, e.g., lines have infinite length. For the purposes of evaluation, unbounded features are bounded by their sample point sets. The resultant bounded test feature is then compared to the reference feature. Evaluation parameters are defined for each type of feature (see [Figure 4.1.1-1](#)).

4.1.2 Evaluation Parameters. Each feature type has a unique set of evaluation parameters. Test results are reported as outlined below. The figures in this section have the following annotation conventions:

- A = angle
- a = cone half-angle
- D = separation distance
- R = reference fit parameter subscript
- r = radius
- t = test fit parameter subscript

4.1.2.1 Line. The test line is bounded by the perpendicular projection of the sample points onto the test line. The (21)
evaluation parameters are (see [Figure 4.1.2.1-1](#))

- (a) the largest separation distance between the bounded test and reference features
- (b) the angle between the test and reference features

4.1.2.2 Circle.

(21)

(a) The test circle is a closed object and naturally bounded. The evaluation parameters are as follows (see [Figure 4.1.2.2-1](#)):

- (1) the absolute value of the difference between the radii of the test and reference circles ($|r_R - r_t|$).
- (2) the distance between the centers of the test and reference circles. This may be a three-dimensional distance.
- (3) the angle between the planes of the test and reference circles, if applicable (see [Table 4.1.2.2-1](#)).

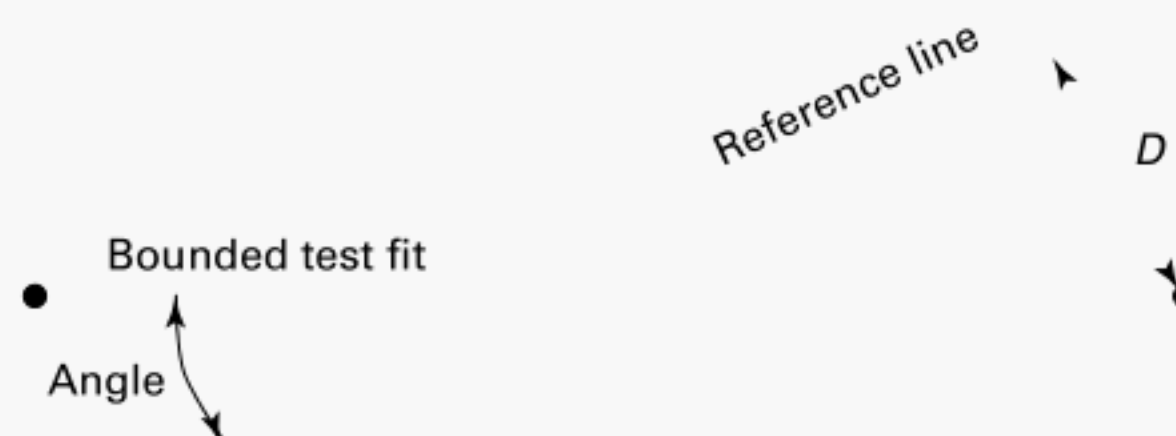
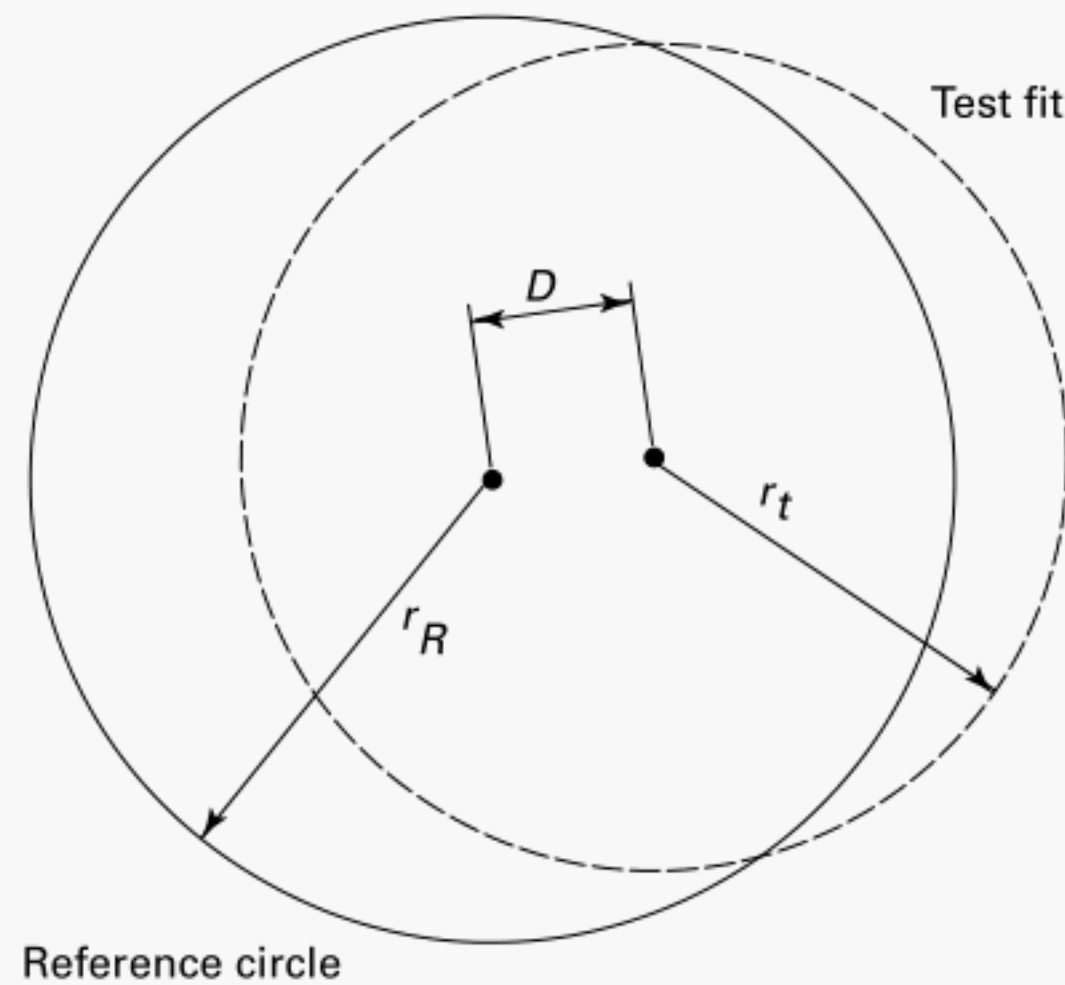
Figure 4.1.2.1-1 Line Evaluation

Figure 4.1.2.2-1 Circle Evaluation

- (b) In the case of least-squares fitting of circles (in three dimensions), the least-squares fit can be defined in two ways.
- (1) The first way is to
 - (-a) fit the points to a least-squares plane
 - (-b) project the points into that plane
 - (-c) fit a circle to the projected points, which is a two-dimensional fit
 - (2) The second way is to define the objective function as the sum-of-squares of the three-dimensional distances from the points to the circle in space; the fit circle is then the one that minimizes that objective function.
- Because of the two possible definitions of the three-dimensional, least-squares fit circle, the test report shall identify which method is used by the software under test, and the reference feature and test results shall be consistent with the definition used.

4.1.2.3 Plane. The test plane is unbounded. Sample points are projected onto the test plane for the evaluation. The evaluation parameters are

- (a) the largest perpendicular distance from the reference plane to any projected sample point in the test plane (see D in Figure 4.1.2.3-1)
- (b) the angle between the test and reference planes

4.1.2.4 Sphere. The test sphere is a closed object and naturally bounded. The evaluation parameters are (see Figure 4.1.2.4-1)

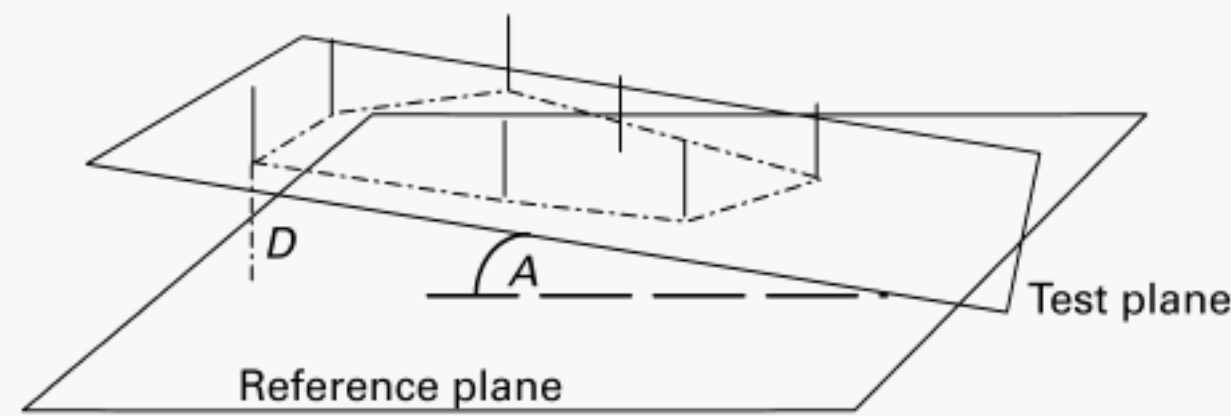
- (a) the absolute value of the difference between the radii of the test and reference spheres ($|r_R - r_t|$)
- (b) the distance between the centers of the test and reference spheres

4.1.2.5 Cylinder. The test cylinder is bounded along its axis by projecting the sample points perpendicularly onto its axis. It is naturally bounded in circumference. The evaluation parameters are

- (a) the absolute value of the difference between the radii of the test and reference cylinders ($|r_R - r_t|$)
- (b) the maximum perpendicular distance from the bounded test cylinder axis to the axis of the reference cylinder (see D in Figure 4.1.2.5-1)

Table 4.1.2.2-1 Circle Fit Types

| Circle Fit Type | Reported Angle |
|---|--------------------------|
| Two-dimensional | N/A (= 0) |
| Three-dimensional, both use same reference plane | N/A (= 0) |
| Three-dimensional, fit plane, then two-dimensional circle | Angle between fit planes |
| Three-dimensional circle fit | Angle between planes |

Figure 4.1.2.3-1 Plane Evaluation

(c) the angle between the axes of the test and reference cylinders (see A in [Figure 4.1.2.5-1](#))

4.1.2.6 Cone

(a) The test cone is bounded along its axis by

(1) projecting the sample data perpendicularly onto the test cone surface

(2) projecting these surface points perpendicularly onto the test fit axis (see [Figure 4.1.2.6-1](#))

It is naturally bounded in circumference. The reference cone axis is similarly bounded.

(b) The cone evaluation parameters are

(1) for each cone, the perpendicular distance from the midpoint of the bounded axis to the corresponding cone surface is computed. The evaluation parameter is the absolute difference between these distances ($|r_R - r_t|$)

(2) the maximum perpendicular distance from the bounded test axis to the unbounded reference axis (see D in [Figure 4.1.2.6-2](#))

(3) the angle between the test and reference axes (see A in [Figure 4.1.2.6-2](#))

(4) the absolute difference between the test and reference included cone half-angles ($|a_R - a_t|$)

4.1.2.7 Evaluation Parameter Summary. [Table 4.1.2.7-1](#) summarizes the evaluation parameters for the seven (21) feature geometries dealt with in this Standard.

In addition to the parameters of [Table 4.1.2.7-1](#), when the objective function is minimum-zone, maximum-inscribed, or minimum-circumscribed, the value of the deviation in the objective function is also reported.

For the cases of maximum-inscribed and minimum-circumscribed objective functions, the deviations in parameters in [Table 4.1.2.7-1](#) (besides the diameter) are typically much larger than the diameter deviations. This is due to the fact that often there are multiple fits that vary little with respect to the objective function (the diameter).

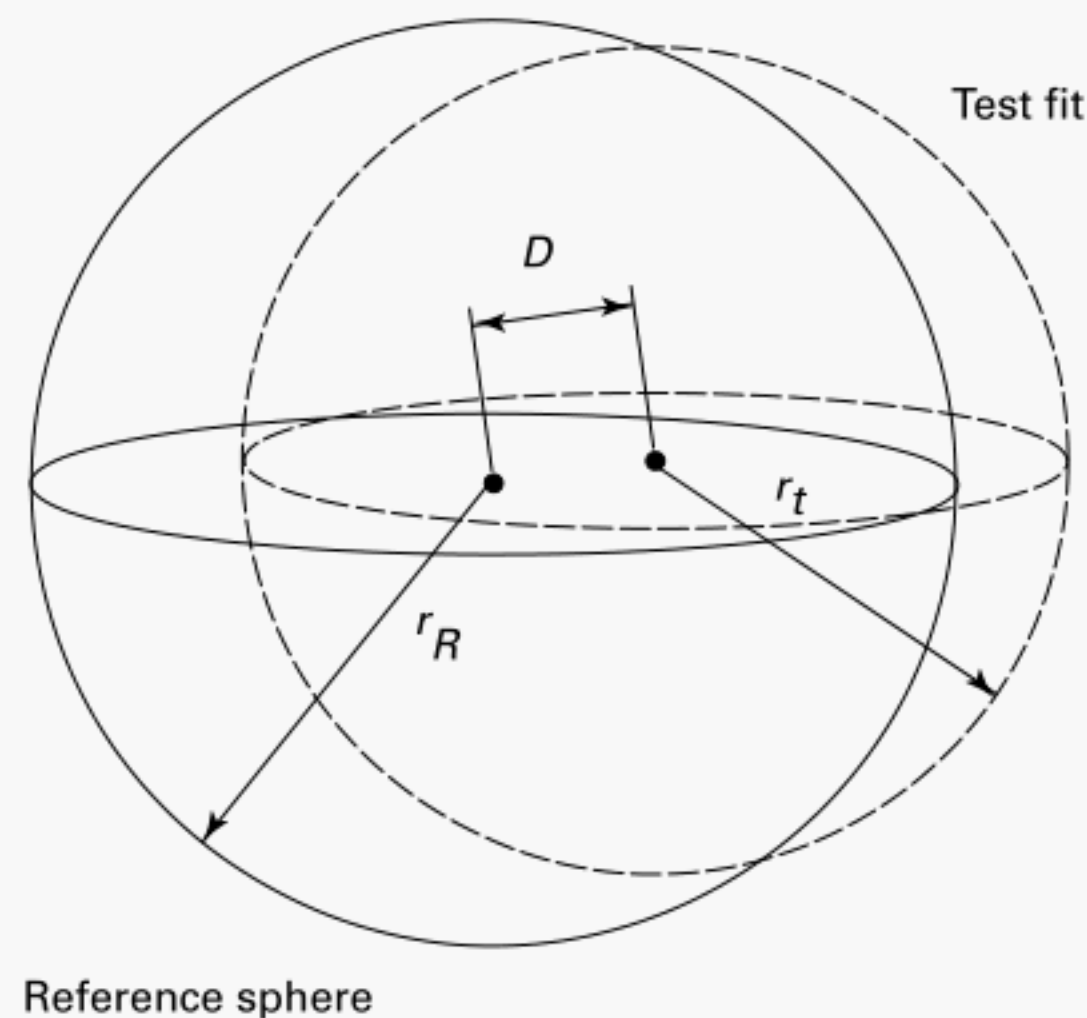
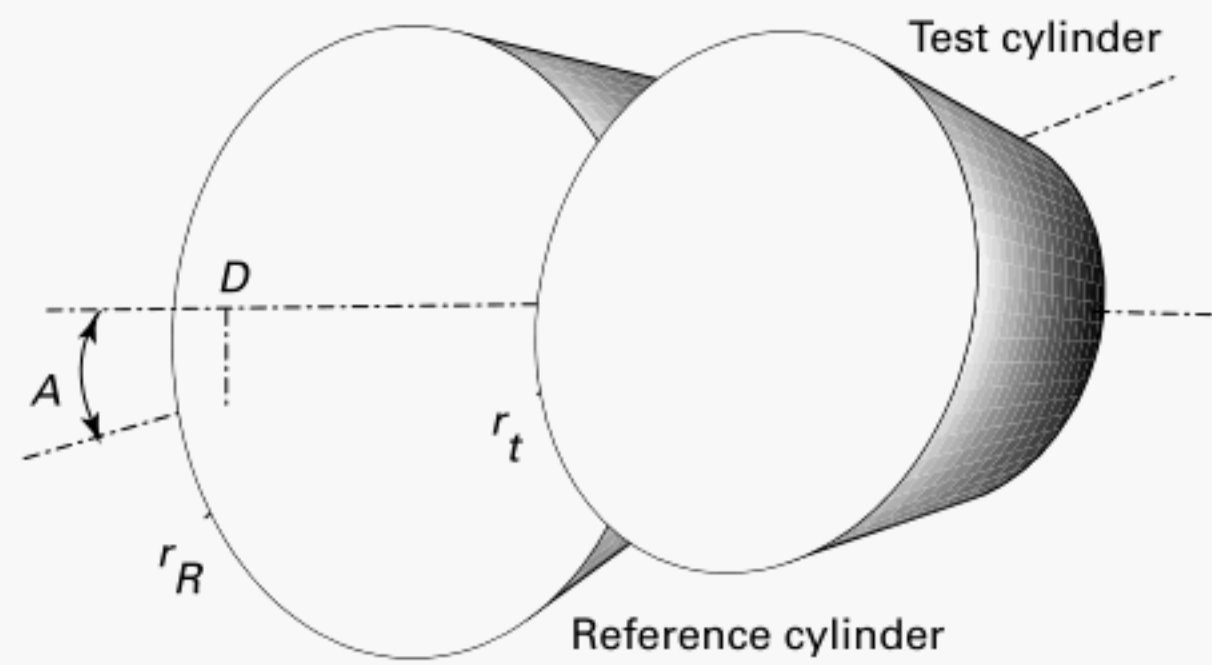
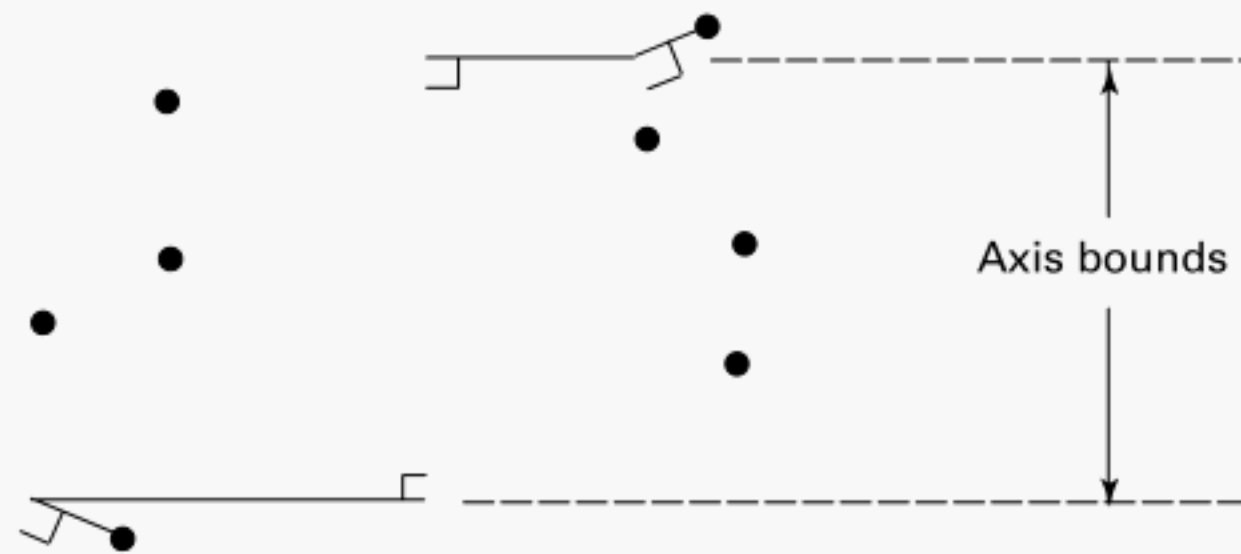
Figure 4.1.2.4-1 Sphere Evaluation

Figure 4.1.2.5-1 Cylinder Evaluation**Figure 4.1.2.6-1 Cone Bounding**

4.2 Characteristics of Robustness

Robustness is the ability of the software to recover from incorrect inputs, such as colinear data points, too few data points, or, for some CMSs, too many data points. When applicable, robustness shall be tested by including incorrect data sets.

4.3 Characteristics of Reliability

Reliability is the ability of the software to resolve a wide variety of problems. The only reliability characteristic to be addressed is the sensitivity of CMS software to variations of input data. See [Nonmandatory Appendix A](#) for information about other factors that affect CMS software performance.

To evaluate CMS software sensitivity, the effects of each factor and interactions among factors should be examined. For each geometric feature type, collections of test data sets shall be designed that include variations in the above factors (see [para. 5.6](#)).

Figure 4.1.2.6-2 Cone Evaluation

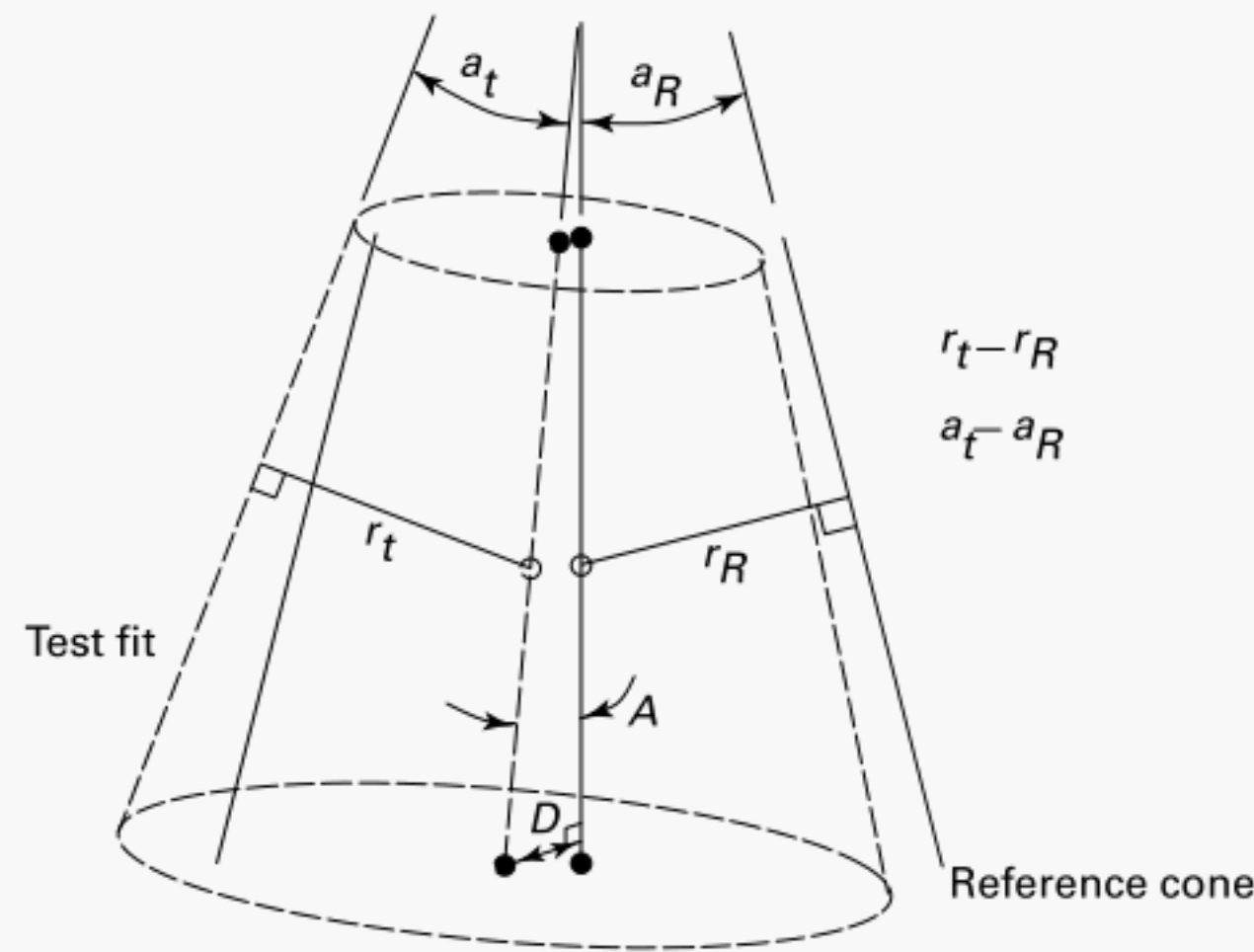


Table 4.1.2.7-1 Evaluation Parameters

(21)

| Feature | Maximum Distance Between | Angle Between | Radii Difference | Cone Half-Angle Difference |
|----------|--------------------------------------|---------------|------------------|----------------------------|
| Line | Lines | Lines | ... | ... |
| Circle | Centers | Planes | Yes | ... |
| Plane | Projected sample point and reference | Planes | ... | ... |
| Sphere | Centers | ... | Yes | ... |
| Cylinder | Axes | Axes | Yes | ... |
| Cone | Axes | Axes | At axis centers | Yes |

GENERAL NOTE: ... = not applicable to the feature.

4.4 Characteristics of Ease-of-Use

Ease-of-use measures the amount of effort required to use the software, including set-up time, documentation, and structure of the code. This Standard only addresses documentation requirements. Refer to [section 6](#) for more information.

4.5 Related Issues

Software performance may be affected by other factors not included in the performance evaluation. Such factors in the areas of algorithms, computing environment, software implementation, and computational effort are discussed below.

4.5.1 Algorithms. The concept of an *algorithm* is often confused with that of an *implementation* of an algorithm. According to Jackson et al., an algorithm “is a problem solving template that leaves some practical details unspecified. It thus corresponds to a class of computer programs (its implementations) with certain sequences of instructions in each implementation corresponding to the steps of the algorithm.” For the purpose of this Standard, algorithms are distinguished by fit criteria (as described in [section 3](#)) and by the geometric entity as described in this section. It is important to differentiate between performance comparisons of different implementations of the same algorithm and performance comparisons of different algorithms. Consideration must also be given to the mathematical representation of the problem, i.e., the parameters used, which may have significant effects on the reported results. Strictly speaking, this Standard is concerned with testing the behavior of algorithm implementation, not the testing of algorithms themselves.

4.5.2 Computing Environment. A single implementation of an algorithm may perform differently in various computing environments. The following factors may affect software results:

- (a) processor characteristics, such as precision and word length
- (b) computer architecture

- (c) operating system
- (d) compiler

4.5.3 Software Implementation. The method of implementing an algorithm may affect its speed and efficiency. Some factors that contribute to this are

- (a) programming language
- (b) use of data structures
- (c) storage requirements

4.5.4 Computational Effort. The effort or time required to compute the results is affected by the three previous factors. Excessive computing effort can adversely affect throughput of a CMS.

5 TEST METHODOLOGIES

This section establishes the general principles, procedures and practices for testing the performance of CMS software.

5.1 Test Principles

For the purposes of this Standard, CMS software is evaluated strictly in terms of its intended function. No assumptions are made regarding the internal structure or operation of the software. The software is subjected to variations of inputs while its outputs are evaluated with respect to a specified objective.

CMS software is tested by the input of sets of test data that reflect the expected range and variability of actual data. Such testing cannot guarantee that the software is completely error free because exhaustive functional testing is impossible.

5.2 Apparatus

The apparatus shall be a testing system interacting with the software under test through exchange of data sets and fit results. Software under test shall be executed in the computing environment in which it will be used. Modifications of the software under test, if any, are limited to those necessary to input the supplied data sets and to extract the fit results. Such modifications shall not change the fit results from what would be produced by the software under test when presented the same data in a production environment.

The data formats, data resolution, and related characteristics of the test must be defined prior to its execution. The person operating the software under test shall be trained in the operation of that software to the extent necessary to input the data, run the software, and gather the output (fit results) in the required format. [Figure 5.2-1](#) illustrates the following major components of a functional CMS software testing system:

- (a) a Reference Pair Generator (RPG) capable of producing reference pairs of data sets and fit results, for specified feature types with controlled range and variability
- (b) a means to transfer data sets to, and receive fit results from, the software under test
- (c) a comparator designed to compare the results of the software under test to the reference results with respect to the objective function and generate an appropriate report

5.2.1 Reference Pair Generation and Validation

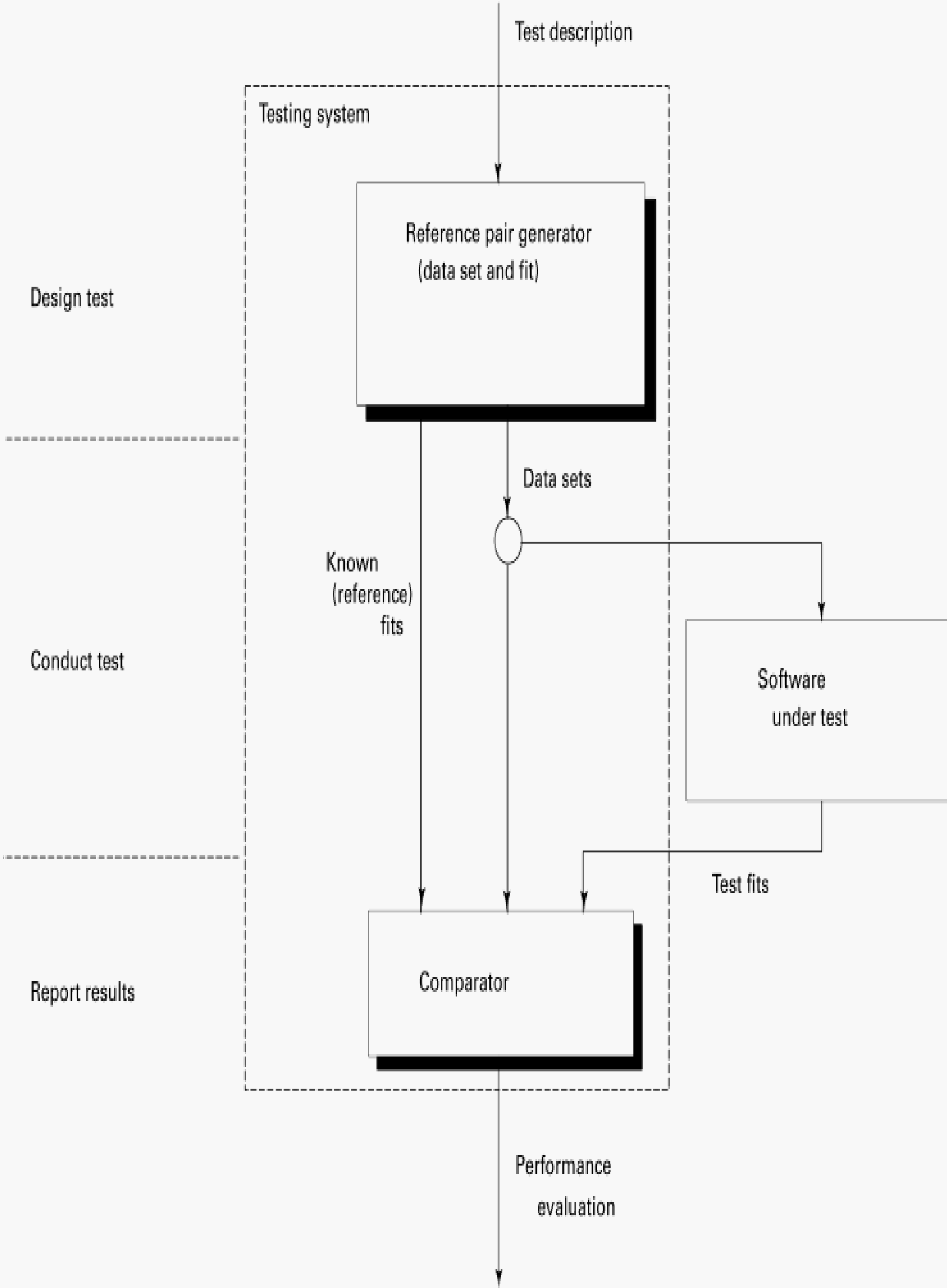
5.2.1.1 Reference Pair Generation. The RPG must be capable of creating a data set and fit. This may be done in two ways.

- (a) A predefined fit result is processed to produce a data set that meets the fit criteria.
- (b) A data set is generated that approximates the feature, and a reference fit is generated from the data by a reference algorithm.

- (21) **5.2.1.2 Validation Reference Pair.** In either case, there is a question regarding the validation of the RPG since it is also a complex software program. Because it is not feasible by current technology to prove the correctness of a software implementation of an algorithm, the following is recommended.

- (a) The comparator shall evaluate the objective function for each test case. The fit yielding the smaller value for the objective function is by definition the better fit.
- (b) If the result of the software under test is better than that of the RPG, then that case shall be omitted from the test report. Information sufficient to describe the test case and the reason for omission shall be reported.
- (c) The agency responsible for the maintenance of the RPG should be notified of any such omitted cases so that appropriate action can be taken.

Figure 5.2-1 Major Components of a Software Testing System



5.3 Test Procedure

5.3.1 Software Performance Evaluation. CMS software performance evaluation shall include the following steps:

- (a) Obtain/generate test data with the testing system.
- (b) Process data with software under test.
- (c) Collect the outputs of the software under test and reparameterize.
- (d) Evaluate the fit results produced by the software under test with the testing system.

5.3.2 Evaluation of Fit Results The following is a generic method for the evaluation of fit results:

- (a) The substitute feature is bounded by the sample point set.
- (b) A set of parameters for the reference feature is provided.
- (c) Evaluation parameters are computed for each pair of test and reference features.
- (d) Summary statistics for the evaluation parameters are computed.

5.4 Input Parameters

All input parameters shall appear on the test report.

5.4.1 Units. The unit of length shall be agreed upon before the test. The unit of angular measure shall be decimal degrees. All input data and test results shall be reported in the agreed upon units.

5.4.2 Maximum and Minimum Size. The maximum size L_{\max} and minimum size L_{\min} define the range of feature sizes for test data set generation. The ratio L_{\max}/L_{\min} shall be no greater than 10^4 . Greater ranges can be accounted for using more than one test, each satisfying this range requirement.

5.4.3 Farthest Position. The farthest distance from the origin that a feature can be placed shall be specified and indicated in the test report. This distance shall be at least $2L_{\max}$.

5.4.4 Types of Features. The input parameters will define the types of features to be evaluated from the set of features supported by this Standard. Only features supported by the CMS may be evaluated.

5.4.5 Maximum Number of Sample Points. The maximum number of sample points used for test generation shall be agreed upon before the test and shall be indicated on the test report.

- (21) **5.4.6 Test Data Precision.** The number of digits to which the test data are generated shall be agreed upon before the test but shall be at least as numerically precise as $10^{-5}L_{\min}$. This does not and should not restrict the number of digits to which fits are computed. For the purposes of software testing, the input data should be thought of as exact, having infinite trailing zeros.

- (21) **5.4.7 Seed Values.** CMS software may require seed values. These values are typically defined by the first few sample points, i.e., a cone seed may require three points for a smaller circle followed by three points for a larger circle. If the test data are constructed to provide such seed values, it shall be noted on the test report for each feature type. Any similar requirements of the software under test that are identified in the software documentation as required for its usage and are provided during the test shall also be noted in the test report. If the software requires the point ordering to not be randomized (as explained in [para. 5.5.5.1](#)), this shall also be identified in the test report.

5.4.8 The Default Test. The default test is defined by the following default input parameters: units in millimeters; $L_{\min} = 1$, $L_{\max} = 500$, farthest position = 1 000, maximum number of sampled points = 500. The test data sets shall be generated to a precision of 10^{-5} .

5.5 Generation of Test Data

For each feature type, 30 data sets shall be generated via computer simulation satisfying the following requirements for size, position, orientation, number of sampled points, sampling plans, and form errors.

- (21) **5.5.1 Sizes.** The sizes of features shall be bounded by L_{\min} and L_{\max} . The 30 sizes shall be determined within three size categories as follows: Generate ten random numbers in each range of $(0, \frac{1}{3})$, $(\frac{1}{3}, \frac{2}{3})$, and $(\frac{2}{3}, 1)$. For each random number x , define the size of the feature as $L_{\min}^{1-x}L_{\max}^x$. The size parameters for the feature types are defined as follows:

| Feature Type | Size |
|--------------|---|
| Line | Bounded length |
| Plane | Maximum of length and width of the bounding rectangle |
| Circle | Diameter |
| Sphere | Diameter |
| Cylinder | Maximum of diameter and bounded height |
| Cone | Maximum of (larger) base diameter and bounded height |

For example, when generating lines with $L_{\min} = 1$ and $L_{\max} = 1\,000$, three size scales would be created by using the above generation scheme. Ten line segments would have sizes between 1 and 10 units, ten would have sizes between 10 and 100 units, and ten would have sizes between 100 and 1 000 units.

Lines and circles can be tested in two- or three-dimensions. A two-dimensional line or circle is restricted to be parallel to a coordinate plane.

The aspect ratio of planes, the height-to-diameter ratio for cylinders, and the height-to-base-diameter ratio for cones shall be between 0.02 and 10. Specifically, in each size category, one ratio shall be 0.02, two shall be 0.1, two shall be 0.3, two shall be 1, two shall be 3, and one shall be 10. For each size category, the degree measures of the apex half-angles ψ for cones shall fall into the ten intervals defined by these 11 values: $1/2$, 1, 2, 4, 10, 20, 30, 40, 60, 70, and 75.

5.5.2 Positions. Test cases shall include data points from some features having centers of mass (centroids) near the origin and some far from the origin, close to the specified farthest position. Feature positions are not restricted to the first quadrant or octant unless such is a special input restriction for the software under test; in which case, the restriction shall be noted on the test report. (21)

5.5.3 Orientations. In each size category, the test cases shall include nominal orientations that are aligned with each of the coordinate axes of the data and one aligned with a vector whose direction is (1, 1, 1). Except for these, orientations shall be determined randomly.

5.5.4 Numbers of Points. For each feature type, one data set in the middle size category shall be comprised of the minimum number of points shown below. Also, one data set shall be comprised of the specified maximum number of points. The remaining data sets shall be comprised of numbers of points strictly between these minimum and maximum values, chosen using a logarithmically random generator. For surfaces, the number of points (above the minimum) may be rounded off to a convenient composite number suitable for a grid pattern, provided the number of points is still strictly between the minimum and maximum values. The minimum numbers of points are as follows: (21)

| Feature | Number of Points | Feature | Number of Points |
|---------|------------------|----------|------------------|
| Line | 2 | Sphere | 4 |
| Plane | 4 | Cylinder | 6 |
| Circle | 3 | Cone | 9 |

5.5.5 Sampling Plans

(21)

5.5.5.1 Distribution. The points in each data set shall be nominally regularly spaced. Even though the points are regularly spaced, the order of the points in each data set shall be randomized. Exceptions to this shall be noted as described in [para. 5.4.7](#). In the cases of cylinders and cones, some distributions lead to multiple solutions. Two parallel rings of three points each can yield two correct, orthogonal fits. Eight points distributed on the corners of a box yield three correct, orthogonal fits. Care must be taken to avoid distributions that are close to these ambiguous cases. This may be ignored when seed values are used to establish approximate orientation.

5.5.5.2 Partially Sampled Surfaces. Surfaces may be partially sampled, representing cases where the entire feature is not accessible or incomplete, e.g., a bearing face or a surface patch of a taper.

Sampled arcs of circles, cones, and cylinders shall be 90 deg, 180 deg, and 360 deg. In each of the three size categories, two data sets shall represent 90-deg samples, two 180-deg samples, and the remaining six 360-deg samples.

Spheres shall be sampled over 90-deg and 180-deg polar patches and an equatorial band defined by a ± 15 -deg angle (30-deg total) from the center. In each of the three size categories, three data sets shall be sampled over 90-deg polar patches, five over 180-deg polar patches, and two over equatorial bands.

For maximum-inscribed and minimum-circumscribed objective functions, the test data sets shall be more fully sampled. Thus, sampling shall cover more than 180-deg patches for circles, spheres, and cylinders.

Table 5.5.6-1 Number of Required Form Errors

| | Line | Plane | Circle | Sphere | Cylinder/Cone |
|---------------------------------|--------------------|---|----------------------|---|---|
| One-dimensional sine [Note (1)] | 4 (0.5, 1, 2,3) | ... | 5 (1, 2, 3, 4, 5) | ... | 1 (0.5) axis sine |
| Surface sine [Note (1)] | ... | 4 (0.5, 0) (0.5, 1) (1, 1) (3, 1) | ... | 6 (0, 0.5) (0, 1) (2, 1) (3, 1) (2, 0.5) (3, 2) | 5 (0, 0.5) bow (0, 0.5) hourglass (2, 0) 2-lobed (3, 0) 3-lobed (3, 1) combination |
| Step | 1 | 1 | 2 | ... | 1 (radial step about axis) |
| Bend | 2 | 2 | ... | ... | ... |
| Taper | ... | ... | ... | ... | 1 |
| Random | 2 | 2 | 2 | 3 | 1 |
| None | 1 | 1 | 1 | 1 | 1 |

NOTE: (1) Frequencies in parentheses.

- (21) **5.5.6 Form Errors.** One data set in each size category shall have no form error (the one in the middle size category using the minimum number of points). When applicable, these three data sets shall not coincide with extreme values of aspect or height-to-diameter ratios or extreme values of a cone's apex half-angle. The remaining test cases shall include a maximum peak-to-valley form error of either 0.1% (four test cases per size category) or 2% (five test cases per size category) of the feature's length scale. The length scales for the feature types are defined as follows:

| Feature Type | Size Parameter |
|--------------|---|
| Line | Bounded length |
| Plane | Minimum of length and width of the bounding rectangle |
| Circle | Diameter |
| Sphere | Diameter |
| Cylinder | Minimum of diameter and bounded height |
| Cone | Minimum of base diameter and bounded height |

The number and type of required form errors for each size category for each feature type are identified in [Table 5.5.6-1](#); their mathematical definitions are given in [Mandatory Appendix I](#). Each form error identified shall coincide with a 2% form error at least once.

In addition to these errors, uniform random errors shall be superimposed as follows:

(a) If the maximum peak-to-peak error was 2% of the feature's length scale, then a three-dimensional, uniformly random error of size 0.1% of the feature's length scale shall be added.

(b) If the maximum peak-to-peak error was 0.1% of the feature's length scale, then a three-dimensional, uniformly random error of size 0.01% of the feature's length scale shall be added.

(c) Lines and circles can be tested as two- or three-dimensional features. When testing as a two-dimensional feature, these random errors shall be restricted to the plane of the feature.

5.6 Test Set

At a minimum, data sets described in [para. 5.5](#) shall be generated. Additional tests may be run to uncover specific problems if required. The guidelines for test generation shall be followed except where they violate stated CMS vendor specifications. Where such exceptions occur, they shall be noted on the test report.

From the nominal feature, a sample set is generated using the guidelines in [para. 5.5](#).

5.7 Process Data With Test Software

(21)

Special conversion software or a modified version of the CMS software may be required to allow for the introduction of data not acquired through the normal CMS data input channel. If the CMS is capable of executing a stored program, a program that performs data set evaluations must be written. This program may be subsequently used to evaluate new versions of CMS software. CMS systems without stored program capability may be manually controlled to perform their evaluations, but it is recommended that automatic methods be made available if possible.

The order of the points in the data set may be changed to satisfy any special requirements of the software under test. If reordering of the data is required, it shall be noted on the test report.

The results of the algorithms should be output in a format compatible with the comparator function.

5.8 Calculation and Interpretation of Results

(21)

The guidelines for algorithm comparison in [para. 4.1](#) shall be used to compare the results of the software under test to the reference results for each data set. For each geometric feature type, a statistical analysis shall be performed to evaluate the root-mean-square (RMS) and maximum magnitude of the observed evaluation parameter values.

Difference angles are to be expressed in microradians. Distance and radii differences shall be converted to the normal units of the CMS (see [para. 5.4.1](#)).

5.9 Reporting of Test Results

A test report shall be produced at the conclusion of the comparison phase. The test report shall include the following information:

- (a) the reference software used and its version identifier
- (b) the characteristics of the software that was tested (including computing environment, software version, and any other necessary identifying characteristics)
- (c) the geometric feature types tested
- (d) any reordering of the data or seed values
- (e) the range of conditions represented by the test data for each geometric feature type
- (f) the RMS value of each evaluation parameter for each geometric feature type
- (g) the maximum observed value of each evaluation parameter for each geometric feature type
- (h) the criteria for identifying bad fits for exclusion from the statistical analysis
- (i) the test results for bad fits excluded from the statistical analysis and the corresponding test data characteristics

If no fits were excluded from the analysis, the RMS statistic includes the effects of both systematic and random deviations between the software under test and the reference results. Thus, it can be interpreted as the expected deviation from true value for the software under test, over the range of conditions represented by the test data. To support this interpretation, the effects of uncertainty inherent in the reference results must be included in an uncertainty statement for the RMS statistic.

If any test results were excluded from the analysis, the above interpretation of the results does not hold. Rather, the software is unreliable for the conditions of the test. Although there is no consistent metrology interpretation of the test results in this case, the results have diagnostic value.

If the default test is used, the following minimum values shall be used where applicable in the test report when reporting RMS or maximum observed values for evaluation parameters:

| | |
|-----------|-------------------------|
| Distances | 10^{-5} μm |
| Angles | 10^{-7} arc sec |

In the case the RMS or maximum value of an evaluation parameter is below the minimum, the reported value shall be reported as "less than 10^{-5} " or "less than 10^{-7} ," as appropriate, along with the corresponding units.

5.10 Periodic Reverification

(21)

CMS software should be evaluated when an upgraded version is released, when there is any change in the computing environment that might affect the results, or when results reported by the software appear to be abnormal.

6 SOFTWARE DOCUMENTATION

This section provides guidelines for minimum documentation for coordinate metrology software.

6.1 Purpose

The purpose is to provide guidelines for preparation of user documentation by CMS manufacturers that will provide, to the software users, a sufficient understanding of the intent and underlying principles of each software procedure used in the analysis of coordinate data.

6.2 Compliance

Coordinate metrology software meets the minimum documentation requirements of this Standard if the guidelines listed under [para. 6.3](#) are followed. The guidelines are for content only, not for format or structure. These guidelines apply to each procedure, or set of procedures, that are applied to a specific dimensioning and tolerancing call-out. For an example, reference [Nonmandatory Appendix B](#).

6.3 Required Information

The information listed below is the minimum required for proper documentation.

6.3.1 Procedure Name. A name used to designate each algorithm implementation.

6.3.2 Brief Description. A one-line description of the procedure.

6.3.3 Standards Compliance. Compliance with applicable gaging standards should be included in this section.

6.3.4 Explanation of Procedure. A detailed description of the procedure and services should be provided as follows.

6.3.4.1 Intent. A concise discussion of the intent of the procedure(s) should be provided. This discussion should address all aspects of the procedure, including input data, calculations and other data processing, and method of part evaluation with respect to tolerance requirements.

6.3.4.2 Underlying Principles. Any underlying principles that the user may need to understand in order to properly use the procedure (to the extent that such knowledge may not be assumed for a skilled operator in general).

6.3.4.3 Illustrated Examples. An illustrated example that describes the relationship of output to input should be provided for each procedure that applies to a specific dimensioning and tolerancing evaluation. This example should graphically show sufficient data points, tolerance zone, and the results in relation to the tolerance zone. If a numerical result is calculated and compared to the tolerance zone, then this result should be graphically displayed. Any applicable datum feature(s) and their relationship(s) to the tolerance zone should be illustrated. The illustration must clearly show the intent of the procedures in relation to the individual data points.

(21) **6.3.4.4 Limitations and Precautions.** Limitations of the procedure and other precautions to the user should be provided.

6.3.5 Input. Descriptions, formats, and examples of the access to the procedure(s) should be provided.

6.3.5.1 Defaults. Default input(s) should be provided.

6.3.5.2 Required Inputs. A description of all required input(s) to the procedure(s) should be provided.

6.3.5.3 Optional Inputs. All optional input(s) should be defined.

6.3.5.4 Interface Equivalence. Description, formats, and examples of all equivalent input statement(s) should be listed for any supported interface.

6.3.5.5 Input Limitations. Known limitations and constraints on the procedure(s) input should be listed, e.g., minimum and maximum number of coordinate points the procedure can process.

6.3.6 Output. Descriptions, formats, and examples of the outputs of the procedure(s) should be provided.

6.3.6.1 Defaults. Descriptions, formats, and examples of default output(s) should be provided.

6.3.6.2 Optional Outputs. Descriptions, formats, and examples of optional output(s) should be provided.

6.3.6.3 Interface Equivalence. Equivalent output statement(s), format(s), and example(s) for any supported interface should be listed.

(21) **6.3.6.4 Output Limitations.** Known limitations of output should be provided, e.g., a limited number of digits reported in output.

6.3.7 Exception Conditions. Listing and definition of the various exception handling procedures should be provided.

6.3.8 Computational Uncertainty. A value characterizing the expected uncertainty contributed by the software (21) should be provided. The value should include the estimated cumulative effects of all computational factors that affect geometric uncertainty, including numerical rounding, convergence criteria used in estimation algorithms, and other factors independent of specific measurement tasks. This value should be one, with the understanding that some applications may have errors that exceed the stated quantity. Reference [para. 4.5](#) for information on related issues. This value does not include the variations that could be observed between various fit objectives (e.g., least squares versus minimum zone), as different fit objectives correspond to different tests.

6.3.9 Associated Datum Features. Reference to datum features documentation (if applicable) should be provided.

MANDATORY APPENDIX I

MATHEMATICAL DESCRIPTIONS OF FORM ERRORS

To describe the form errors, a perfect, nominal feature is first described, having a convenient location and orientation. The form errors are then described in this position, as well as a description of the form error. These features would be translated and rotated in the actual test.

(a) Nominal Features

- (1) *Line*. A line segment having endpoints $(0, 0, 0)$ and $(L, 0, 0)$.
- (2) *Plane*. A rectangle having corners $(0, 0, 0)$, $(L, 0, 0)$, and $(0, W, 0)$.
- (3) *Circle*. A circle in the x - y plane centered at the origin, defined in polar coordinates by $r = R$.
- (4) *Sphere*. A sphere centered at the origin, defined in spherical coordinates by $\rho = R$.
- (5) *Cylinder*. A truncated cylinder defined in cylindrical coordinates by $r = R$ and having extent from $z = 0$ to $z = h$, where h is the height of the cylinder.

(6) *Cone*. A frustum defined in cylindrical coordinates by $r = R + z \sin \psi$ and having extent from $z = 0$ to $z = h$, where h is the cone's height, and ψ is the cone's apex angle.

Let A denote the desired amplitude of the error.

(b) 1-D Sine Errors of Frequency ν

- (1) *Line*. $z = A \sin(2\pi x \nu / L)$.
- (2) *Circle*. $r = R + A \sin(\nu \theta)$ expressed in polar coordinates.
- (3) *Cylinder and Cone*. Points are shifted from the nominal in the x -direction by an amount $A \sin(2\pi x \nu / L)$.

(c) Surface Sine Errors of Frequencies ν_1, ν_2

- (1) *Plane*. $z = A/2 [\sin(2\pi x \nu_1 / L) + \sin(2\pi y \nu_2 / W)]$.
- (2) *Sphere*. $\rho = R + A/2 [\sin(\nu_1 \theta) + \sin(\nu_2 \psi)]$ expressed in spherical coordinates.
- (3) *Cylinder*. $r = R + A/2 [\sin(\nu_1 \theta) + \sin(2\pi z \nu_2 / h)]$ expressed in cylindrical coordinates.
- (4) *Cone*. $r = R + z \sin \psi + A/2 [\sin(\nu_1 \theta) + \sin(2\pi z \nu_2 / h)]$ expressed in cylindrical coordinates.

For the "hourglass" form error for cylinders and cones, replace $2\pi z \nu_2 / h$ with $(\pi + 2\pi z \nu_2 / h)$ in the preceding two equations. (ν_2 would be 0.5 in these cases.)

If $\nu_1 = 0$ or $\nu_2 = 0$, replace $A/2$ with A in the above equations.

(d) Step Errors

- (1) *Line*. If $x > x^*$, $z = A$, else $z = 0$; x^* is chosen randomly between $L/4$ and $3L/4$.
- (2) *Plane*. If $ax + by + c > 0$, then $z = A$, where $ax + by + c = 0$ defines a line (in the x - y plane) chosen randomly but passing through the rectangle having corners $(L/4, W/4, 0)$, $(3L/4, W/4, 0)$, and $(L/4, 3W/4, 0)$.
- (3) *Circle*. If $0 \leq \theta \leq \theta^*$, then $r = R + A$, where θ^* is chosen randomly between 90 deg and 180 deg.
- (4) *Cylinder*. If $0 \leq \theta \leq \theta^*$, then $r = R + A$, where θ^* is chosen randomly between 90 deg and 180 deg.
- (5) *Cone*. If $0 \leq \theta \leq \theta^*$, then $r = R + z \sin \psi + A$, where θ^* is chosen randomly between 90 deg and 180 deg.

(e) Bend Errors of Angle α

- (1) *Line*. If $x > x^*$, then $z = (x - x^*) \tan \alpha$, else, $z = 0$; x^* is chosen randomly between $L/4$ and $3L/4$.
- (2) *Plane*. If $ax + by + c > 0$, then $z = (ax + by + c) \tan \alpha$, where $ax + by + c = 0$ defines a line (in the x - y plane) chosen randomly but passing through the rectangle having corners $(L/4, W/4, 0)$, $(3L/4, W/4, 0)$, and $(L/4, 3W/4, 0)$.

(f) Taper of Angle α

- (1) *Cylinder*. If $z > z^*$, then $r = R + (z - z^*) \tan \alpha$; else $r = R$, where z^* is chosen randomly between $h/4$ and $3h/4$.
- (2) *Cone*. If $z > z^*$, then $r = R + z \sin \psi + (z - z^*) \tan \alpha$; else $r = R + z \sin \psi$, where z^* is chosen randomly between $h/4$ and $3h/4$.

NONMANDATORY APPENDIX A

FACTORS THAT INFLUENCE THE RESULTS

A-1 FACTORS OF SOFTWARE AND COMPUTATIONAL ENVIRONMENT

(21)

The following factors affect the quality of computations carried out by CMS software:

(a) *Feature Geometry.* CMS software behavior may be affected by a feature's geometry, notably its size and location. Depending upon data manipulation techniques employed, software may be less reliable for features of large size or features located far from the origin.

(b) *Feature Form Error.* Errors of form (straightness, roundness, cylindricity, etc.) of measured features affect the calculations of position, size, and orientation by software.

(c) *Feature Sampling Strategy.* The number of sampled points and the pattern in which those points were taken may affect CMS software reliability. In most cases, the mathematical minimum number of points necessary to determine a geometric element is not sufficient for the measurement of an actual feature. Strategies of point density and pattern sampling can be found in BS 7172-1989.

(d) *Point Measurement Error.* Errors in each sampled point that were induced by the point measurement process may affect the reliability of CMS software. However, this issue is beyond the scope of this Standard; see the ISO Guide to the Expression of Uncertainty in Measurement for information about the propagation of errors through calculations.

A-2 FACTORS OF IMPLEMENTATION

(21)

The output accuracy of a CMS is also influenced by a combination of factors beyond the influences of software and the computational environment. The CMS user should be aware of these factors and make every effort to control their influence. These factors, which partially overlap with [section A-1](#), include

(a) the sampling strategy on a feature geometry having a particular form error (straightness, roundness, cylindricity, etc.). Strong interactions between form error and sampling strategy are likely. Strategies of point density and pattern sampling can be found in BS 7172-1989 and ISO 14406:2010.

(b) the accuracy characteristics of the coordinate data, as determined by proper verification. (For many CMS technologies, standards exist describing verification tests.)

(c) the physical environmental effects on the CMS and workpiece

(d) the effects of the use of substitute geometry by the CMS software and the resulting uncertainty when measuring geometric features

(e) the factors that affect the sensitivity and behavior of the algorithms, including

(1) point measurement errors on imperfect surfaces caused by less than the minimum number of points (point density) needed to identify a feature

(2) sampling errors on imperfect surfaces resulting from poor placement or inadequate coverage of the characteristic being sampled

(3) workpiece form or positional errors caused by improper measurements and the variables introduced by the mathematics

A-3 FACTORS OF ALGORITHM SELECTION

Software algorithms, like any other tools of manufacturing, may be misused or misapplied. Factors that must be considered in the selection of software for a measurement task include the following:

(a) the choice of the objective function to evaluate a geometric requirement

(b) the use of two-dimensional software to inspect a three-dimensional characteristic does not necessarily allow for required degrees of freedom, e.g., MMC positional tolerances

(c) the CMS part program may not meet the geometric requirements of the workpiece as expressed on the engineering drawing

NONMANDATORY APPENDIX B EXAMPLE DOCUMENTATION

(21)

(21) B-1 GENERAL INFORMATION

This Appendix presents an example of acceptable documentation. The example is not necessarily acceptable measurement practice.

DISCLAIMER: The sole purpose of this example is to demonstrate adequate documentation practice and should not be construed as explicitly or implicitly endorsing or requiring any single method of calculation, input, output, illustration, etc. A hypothetical brand CMM, XCMM with a native language XMML is used in the following example.

In this example, 15 points have been measured on a surface and assigned to a set called PLANE1 and are to be evaluated against a tolerance of 0.010 mm.

B-2 PROCEDURE NAME

The procedure name is *flatness*.

B-3 BRIEF DESCRIPTION

This procedure calculates the flatness of a plane.

B-4 STANDARDS COMPLIANCE

Calculations of flatness comply with the following standards: Standard XXX and Standard YYY.

B-5 EXPLANATION OF PROCEDURE

To calculate the flatness of a geometric plane, using data points that are a sample of the surface, which approximates the plane, and then evaluate it against a tolerance value.

B-5.1 Intent

A least-squares plane is calculated from the measured points assigned to the set PLANE1. The distances between the least-squares plane and the two extreme points on each side of this plane is calculated, e.g., 0.0011 on one side and 0.0022 on the other. These distances are added with the result being the calculated flatness value, e.g., 0.0033. This calculated difference is compared to the tolerance (0.010 – 0.0033).

B-5.2 Underlying Principles

To find an ideal plane, the sum of the squares of the normal distances from each point to the plane is a minimum. Once this plane is determined, the farthest point on each side of the plane is resolved. The distance between these two points is calculated, normal to the plane, and identified as the flatness.

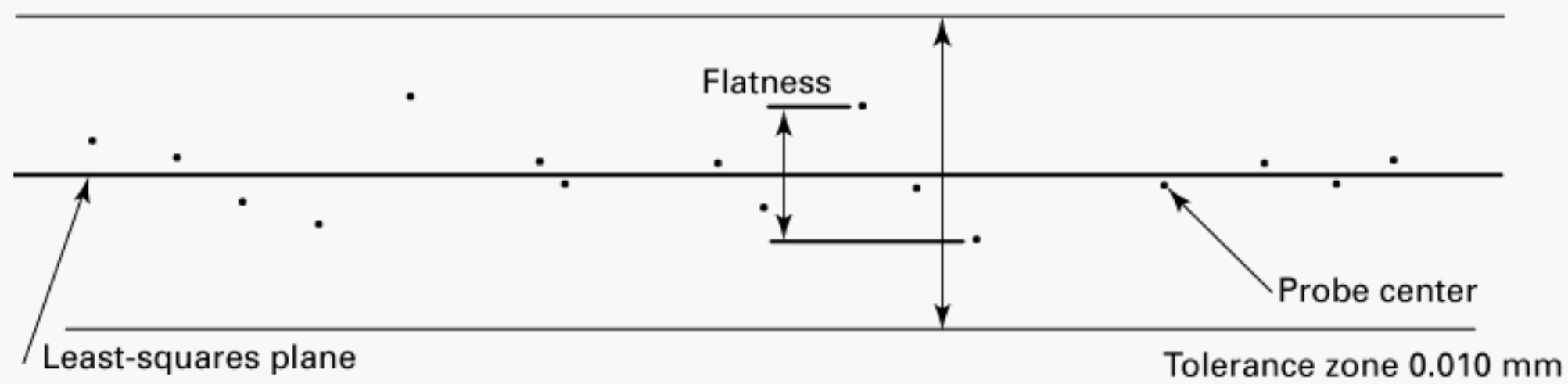
B-5.3 Illustrated Example

See [Figure B-5.3-1](#).

(21) B-5.4 Limitations and Precautions

Flatness procedure can be accessed in the following ways:

(a) pressing the = symbol on the keypad and typing in the name PLANE1. At the prompt, enter the tolerance value of 0.010

Figure B-5.3-1 Flatness Example

(b) type in the XMML command

```
fltns (ele = PLANE1, tol = 0.010)
```

B-6 INPUT

B-6.1 Defaults

(21)

If no tolerance value is entered, the procedure will default to 0.025 mm.

B-6.2 Required Inputs

The name of the set of points (PLANE1 in this case) must be input.

B-6.3 Optional Inputs

A statistics terminal display option is available through the XMML command by adding "sta = term." The resulting command would be

```
fltns (ele = PLANE1, tol = 0.010, sta = term)
```

B-6.4 Input Limitations

(21)

The maximum number of points that can be computed is 9999. The minimum number of points is 6.

B-7 OUTPUT

(21)

The flatness value is printed in the following default format:

```
FLATNS of: $$$$$$ = ##.#### in..... ##.##% of #.#### TOL
```

If the calculated value is greater than the tolerance, the characters OUTOFTOL are printed on the next line. In this case, the calculated flatness is 0.0033, and the output would read

```
FLATNS of: PLANE1 = 0.0033 in..... 33.3% of 0.0100 TOL
```

B-7.1 Defaults

The above is the default format.

B-7.2 Optional Output

An additional optional output format is the statistics. If this option is exercised, a histogram of the individual point deviations are displayed on the terminal but are not printed.

B-7.3 Output Limitations

(21)

The output limits are 4 decimal places (inches) or 3 decimal places (metric).

(21) **B-8 EXCEPTION CONDITIONS**

The CMS system outputs the following error messages when exception conditions occur.

(a) SPATIAL DISTRIBUTION ERROR means that the points are outside the prescribed distribution, indicating that one or both of the following rules were violated:

(1) The thickness must be less than half the width.

(2) The width must be greater than one-tenth the length.

Either remeasure surface taking care not to exceed these rules, or delete points outside of this spatial boundary, and recalculate.

(b) POINT NUMBER MAX means that over 9 999 points have been submitted to the procedure for calculation. Remeasure surface taking 9 999 or fewer points, or delete points until 9 999 remain, and recalculate.

(c) POINT NUMBER MIN means that fewer than six points have been submitted to the procedure for calculation. Remeasure surface taking at least six points.

(21) **B-9 COMPUTATIONAL UNCERTAINTY**

The least-squares fitting software was evaluated in accordance ASME B89.4.10 and found to have an RMS deviation of 10^{-5} mm for plane separation and 0.02 arc sec for plane tilt.

B-10 ASSOCIATED DATUM FEATURES

Flatness is not computed with respect to any other features.

NONMANDATORY APPENDIX C SUBSTITUTE FEATURES

C-1 GENERAL INFORMATION

(21)

This Appendix is directed at the computer programmer concerned with developing substitute feature software.

A substitute feature is a perfect-form geometry (circle, plane, cylinder, etc.) used to represent an actual feature during subsequent part evaluation. A substitute feature is the “representation” of the measured data points. This Appendix describes the most common methods used to define the substitute feature.

Fit criteria lead to an optimization problem, the solution of which defines the parameters of the substitute geometry. With some exceptions, more than one substitute feature may optimize any one criterion. Any application sensitive to such ambiguities must guard against them to ensure proper results.

The mathematical model used in this Appendix is a substitute feature characterized by a vector of parameters b . The perfect-form geometry is defined by a function $f_b(p)$ that assigns a real number to every point p in space. The substitute feature surfaces is described by the equation $f_b(p) = 0$. The entire space is divided into two half spaces by the inequalities $f_b(p) < 0$ and $f_b(p) > 0$. Any particular geometric form can be represented by a wide range of functions f . In this Appendix, the only restrictions on the functional form of f are features of size (i.e., circles, cylinders, spheres, parallel lines, and parallel planes), the half space $f_b(p) < 0$ correspond to the intuitive notion of “inside the feature,” and the half space $f_b(p) > 0$ correspond to the “outside” of the feature. A particular functional form f_b may involve constraints on b to maintain the validity of the representation. Such constraints are not considered in this Appendix, although they should be addressed in a practical implementation of a fitting algorithm.

All the fitting criteria deal with the distance of the measured data points to the substitute feature. If p_i is the i^{th} observed data point, then define

$$e_i(b) \doteq \pm \min_q \{|p_i - q|: f_b(q) = 0\}$$

e_i is the orthogonal distance from the observed point p_i to the surface of the substitute feature. The sign of e_i is chosen to correspond to the sign of $f_b(p_i)$, i.e.

$$\begin{aligned} e_i(b) &> 0 \text{ when } f_b(p_i) > 0 \\ e_i(b) &= 0 \text{ when } f_b(p_i) = 0 \\ e_i(b) &< 0 \text{ when } f_b(p_i) < 0 \end{aligned}$$

It should be noted, that if the feature is of perfect form, there exists a value of b for which $e_i(b) = 0$ for all i . In that event, all of the fitting criteria discussed herein result in the same substitute feature. In practice, this situation may appear to exist when the errors in the actual feature are smaller than the resolution of the measuring device.

C-2 L^P -norm OPTIMIZATION

(21)

The objective for L^P -norm estimation is to determine the parameters of a substitute feature that minimize the sum of the P^{th} power of the absolute deviations between the surface of the substitute feature and the observed values. The L^P -norm estimation problem is defined as finding the values of the feature parameters b that minimize

$$\left(\frac{1}{N} \sum_{i=1}^N |e_i(b)|^P \right)^{\frac{1}{P}}$$

The “best fit” substitute feature is the one that minimizes the L^P -norm.

C-2.1 Least Squares

When $P = 2$, the L^P -norm estimation problem is known as normal least-squares or orthogonal distance regression. The term *least squares* is the usual term in the coordinate metrology community.¹ Least-squares fitting can be formulated as the following optimization problem:

$$\min_b \sum_{i=1}^N e_i^2(b)$$

When the values $e_i(b)$ are linear in b , the L^P -norm estimation problem is also known as the total least-squares problem.

(21) C-2.2 Minimum Zone

When P approaches infinity, L^P -norm estimation becomes minimum zone fitting. Mathematically, as $P \rightarrow \infty$, the optimization becomes

$$\max_i |e_i(b)|$$

Finding the minimum zone fit is finding the parameter b that minimizes the maximum magnitude error. This is sometimes called the two-sided minimax fit. (See [section C-3](#) for one-sided minimax fits.)

The minimum zone fit is often used in applications that require the substitute feature to be as close as possible to the observed data points. This situation can be formulated as the following optimization problem:

$$\min_b \max_i |e_i(b)|$$

(21) C-3 ONE-SIDED L^P -NORM OPTIMIZATION

The objective for one-sided L^P -norm optimization, also called constrained L^P -norm optimization, is to determine the parameters of a substitute feature that minimize the sum of the P^{th} power of the absolute deviations between the surface of the substitute feature and the observed values, while subject to the constraint that the substitute feature does not pass through the material of the part measured (aside from contacting its boundary). The constrained L^P -norm optimization problem is defined as finding the values of the feature parameter b that minimizes

$$\min_b \max_i |e_i(b)|$$

subject to the appropriate material constraint

$$e_i(b) \geq 0, \text{ for all } i = 1, \dots, N$$

or

$$e_i(b) \leq 0, \text{ for all } i = 1, \dots, N$$

C-3.1 Constrained Least Squares

When $P = 2$, the constrained L^P -norm estimation problem is known as constrained least-squares or constrained orthogonal distance regression. Constrained least-squares fitting can be formulated as the following optimization problem:

$$\min_b \sum_{i=1}^N e_i^2(b)$$

subject to either

$$e_i(b) \geq 0, \text{ for all } i = 1, \dots, N$$

when all observed points are to be on the side of the feature corresponding to $f_b \geq 0$, or

¹ It should be noted that outside the field of coordinate metrology, the term *least squares* usually denotes a different objective from the approach presented herein.

$$e_i(b) \leq 0, \text{ for all } i = 1, \dots, N$$

when all observed points are to be on the side of the feature corresponding to $f_b \leq 0$.

ASME Y14.5.1 defines a datum plane using a convex envelope followed by a constrained least-squares optimization. This case leads to a minimization of a weighted sum-of-squares.

C-3.2 One-Sided Minimax

(21)

The one-sided minimax approach is often used in applications that require the substitute feature either to contain every observation point p_i or to contain none of the observation points. This situation can be formulated as the following constrained optimization problem:

$$\min_b \max_i |e_i(b)|$$

subject to either condition as stated in [para. C-3.1](#).

C-4 MINIMUM CIRCUMSCRIBING AND MAXIMUM INSCRIBING METHODS

Alternative circumscribing and inscribing methods exist for features of size. Although these alternative methods appear to be very similar to one-sided minimax methods, they are very different. The objective of the circumscribing method is to minimize the size of the substitute feature while keeping all the observed points p_i inside the substitute feature. Similarly, the objective of the inscribing method is to maximize the size of the substitute feature while keeping all the observed points p_i outside the substitute feature.

The substitute features generated by these methods are usually different from those created by the one-sided minimax methods. However, a relationship does exist between these methods. The size of the inscribed minimax feature is not larger than the size of the largest inscribed feature. Similarly, the size of the circumscribed minimax feature is not smaller than the size of the smallest circumscribed feature.

C-4.1 Minimum Circumscribed

(21)

The minimum-circumscribed feature is determined as a substitute feature that has the smallest size $R(b)$ yet contains all the observed data points. This is the constrained optimization problem, as follows:

$$\min_b R(b)$$

subject to the constraints

$$e_i(b) \leq 0, \text{ for all } i = 1, \dots, N$$

C-4.2 Maximum Inscribed

(21)

The maximum-inscribed feature is determined as a substitute feature that has the largest size $R(b)$ yet contains none of the observed data points. This is the constrained optimization problem, as follows:

$$\min_b R(b)$$

subject to the constraints

$$e_i(b) \geq 0, \text{ for all } i = 1, \dots, N$$

Additional constraints must be added to ensure the range of substitute features considered are reasonable. Without these additional constraints, the maximum inscribed feature is an infinitely large feature with its center or axis infinitely far away from the observed data points.

For example, a circle or a sphere containing observed data points that enclose the desired substitute feature can be stated as requiring the center $c(b)$ of the substitute feature to be inside the convex hull of the observed data points.

$$\begin{aligned}
 c(b) &= \sum_{i=1}^N \lambda_i p_i \\
 \sum_{i=1}^N \lambda_i &= 1 \\
 \lambda_i &\geq 0 \text{ for } i = 1, \dots, N
 \end{aligned}$$

C-5 OTHER APPROXIMATIONS

Some implementations use the representation function values $f_b(p_i)$ directly instead of the distance function $e_i(b)$. When the representation function $f_b(p)$ is not equal to the distance, then the optimization will produce a different resultant than the methods described in the previous sections. For example, the representation formula for a sphere may be

$$f_b(p_i) = (x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2 - r_c^2$$

whereas the distance formula is

$$e_i(b) = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2 + (z_i - z_c)^2} - r_c$$

The use of the representation formula in a least-squares approximation results in finding the parameter vector $b = \{x_c, y_c, z_c, r_c\}$ that minimizes $\sum_{i=1}^N f_b^2(p_i)$. In general, this solution will be different from the least-squares vector b that minimizes

$$\sum_{i=1}^N e_i^2(b) .$$

NONMANDATORY APPENDIX D FUNCTIONAL GAGE SIMULATION

(21)

D-1 GENERAL INFORMATION

(21)

One method of evaluating geometric requirements is to mathematically model a functional gage and calculate whether or not the simulated “gage” will “fit” the part. This is somewhat analogous to a hard gage fitting a part, with allowable relative motion between the gage and part. Often this approach is the only workable solution when both datums and/or multiple features are toleranced with the maximum material condition (MMC) modifiers.

Adequate Functional Gaging simulation requires careful attention to several issues including part representation, gaging process simulation, interpretation of the output, the role of substitute features, and surface sampling methods. In this process, certain precautions must be observed.

D-2 METHODS OF PART REPRESENTATION

Typically, one of three types of analysis is used to represent the “part” calculated from the measurement samples.

D-2.1 Point Method

The coordinate data samples are treated as infinitesimal but real part material. Each sample point is investigated as to whether or not it crosses or “interferes” with the mathematical gage boundaries.

D-2.2 Ideal Substitute Geometry Method

Ideal substitute geometry is calculated for the features under investigation. In order that the gaging principles are not violated, this substitute geometry is usually a maximum-inscribed or minimum-circumscribed circle or cylinder. This substitute geometry may also be a least-squares fit shifted by a statistical multiplier or to the point of extreme material. Intersections of gage and “part” surfaces indicate interference, thereby simulating a No-Go condition of a functional gage inspection.

D-2.3 Higher Order Fitting Method

A higher order surface is fit to the data for each feature to accommodate the variation of form as well as size. Intersection of surfaces indicate interference, thereby simulating a No-Go condition of a functional gage inspection.

D-3 GAGING PROCESS

The actual “gaging” process is usually a process whereby successive iterations attempt to lessen the magnitude of the interferences between the gage and the “part” until no interference is realized. These iterations are relative movements between the gage and the part representations. Degrees of freedom of the movements are constrained by the datum reference call outs.

D-4 OUTPUT AND INTERPRETATION

D-4.1 Go/No-Go

The primary output is an accept/reject disposition.

D-4.2 Maximum Interference

(21)

If the gage cannot fit the part, the maximum interference (after optimization) is usually indicated.

D-4.3 Number of Iterations

The number of iterations required to either fit or determine a no-fit condition are given.

D-4.4 Location of Interferences

Coordinate locations of the interferences are listed.

D-4.5 Possible Scenarios of Rework to Allow “Fitting”

More sophisticated simulations may indicate measures of workpiece rework to improve chances of gage fitting.

D-5 SUBSTITUTE FEATURES

Substitute features may be used in Functional Gaging if they provide adequate information for the analysis. Considerations include sampling density, form error, workpiece tolerances, and type of fit.

D-6 POINT SETS

Point sets should be retained for Functional Gaging analysis. The entire set should lie within the workpiece tolerance zone.

D-7 PRECAUTIONS

D-7.1 Information Extrapolation

Functional Gaging simulation should be used as a tool in conjunction with other data analyses to correctly ascertain workpiece characteristics. Remember its major function is to simulate a Go/No-Go gage.

D-7.2 Error Allowance

Makers and designers of functional gages consider the tolerance in manufacturing the gages and its effect on part acceptability. Just as there are errors in building a hard gage, there are errors in the simulated gaging processes that should always be considered whether or not they are incorporated into the gaging calculations.

D-7.2.1 Measurement System Uncertainty. This error is due to both systematic and random errors of the measuring process and can have a variety of sources.

D-7.2.2 Part Sampling Error. This error is due to the measuring instruments ability to only sample the workpiece surfaces whereas a hard functional gage can contact the full functional surfaces of a part.

NONMANDATORY APPENDIX E

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B89 AMERICAN NATIONAL STANDARDS FOR DIMENSIONAL METROLOGY AND CALIBRATION OF INSTRUMENTS

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| B89 Report-1990 | Parametric Calibration of Coordinate Measuring Machines (Technical Paper) |
| B89.1.2M-1991 | Calibration of Gage Blocks by Contact Comparison Methods (Through 20 in. and 500 mm) |
| B89.1.5-1998 (R2019) | Measurement of Plain External Diameters for Use as Master Discs or Cylindrical Plug Gages |
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